Ex3F06 (2)

Part 1, MULTIPLE CHOICE, 5 Points Each

1 The mean, $\mu = E(X)$ of the probability distribution given below is $\mu = -1$. Which of the following gives the variance $\sigma^2 = Var(X)$?

k	Pr$(X = k)$
-2	.5
-1	.3
1	.1
2	.1

(a) $\sigma^2 = 0.2$ (b) $\sigma^2 = 0$ ~~(c)~~ $\sigma^2 = 1.8$ (d) $\sigma^2 = 14$ (e) $\sigma^2 = 1.5$

$\sigma^2 = \sum (k-\mu)^2 p(x)$

$= (-2+1)^2(.5) + (-1+1)^2(.3) + (1+1)^2(.1) + (2+1)^2(.1)$

$= .5 + 0 + .4 + .9 = 1.8$

2 If Z is a random variable with a standard normal distribution, use the tables provided to find

$$Pr(-1 \leq Z \leq 0.9).$$

(a) .2328 ~~(b)~~ .6572 (c) .8159 (d) .1587 (e) .9746

$Pr(-1 \leq Z \leq 0.9) = Pr(Z \leq 0.9) - Pr(Z \leq -1)$

$= .6572$

1

3 The amount of time it takes (for a person) to run 100m is normally distributed with mean $\mu = 19$ sec. and standard deviation $\sigma = 2$ sec. What is the probability that a person chosen at random can run 100m in less than 16 seconds.

~~(a)~~ .0668 (b) .0228 (c) .9772 (d) .9332 (e) .5793

X = Time person takes to run 100 m

$P(X < 16)$

$= P\left(\frac{X-19}{2} < \frac{16-19}{2}\right)$

$= P(Z < -\frac{3}{2})$ where Z is standard Normal

$= .0668$

4 Consider the matrices

$$A = \begin{pmatrix} -1 & 1 \\ 2 & 0 \\ 1 & 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 3 \end{pmatrix}.$$

Which of the following gives the matrix $2A - B$.

(a) $\begin{pmatrix} 1 & 1 \\ 2 & 1 \\ 1 & 5 \end{pmatrix}$ (b) $\begin{pmatrix} 1 & 0 \\ 2 & 1 \\ 2 & 7 \end{pmatrix}$ (c) $\begin{pmatrix} -4 & 1 \\ 4 & 1 \\ 3 & 11 \end{pmatrix}$ (d) $\begin{pmatrix} -3 & 2 \\ 2 & -1 \\ 0 & 1 \end{pmatrix}$ ~~(e)~~ $\begin{pmatrix} -4 & 3 \\ 4 & -1 \\ 1 & 5 \end{pmatrix}$

$2A - B =$

$$\begin{pmatrix} -2 & 2 \\ 4 & 0 \\ 2 & 8 \end{pmatrix} - \begin{pmatrix} 2 & -1 \\ 0 & 1 \\ 1 & 3 \end{pmatrix} = \begin{pmatrix} -4 & 3 \\ 4 & -1 \\ 1 & 5 \end{pmatrix}$$

2

5 Peter has 2 hours = 120 minutes to complete his math homework before he goes to a basketball game with his friends. On his homework there are 15 short answer questions and 10 long questions (optimization problems!). A long question takes 10 minutes to answer and a short question takes 5 minutes to answer. Peter must do at least 3 long problems and at least 5 of the short problems. The long questions are worth 5 points each and the short questions are worth 3 points each. Let x denote the number of long questions that Peter decides to do and let y denote the number of short questions that Peter decides to do, which set of constraints and objective function given below describe the optimization problem that Peter must solve (assuming that he wants to gain the maximum number of points possible). The following table might help:

	Long Questions (x)	Short Questions (y)	
Time	10	5	120 min
Required to do	3	5	
Given	10	15	
Points	5	3	

$10x+5y \le 120$
$x \geq 3 \quad y \geq 5$
$x \le 10 \quad y \le 15$
Objective
maximize
$5x+3y$.

(a)
$$\begin{gathered} 10x + 5y \leq 120 \\ x \geq 3, \quad y \geq 5 \\ x \leq 10, \quad y \leq 15 \\ \text{Objective function: } 15y + 10x \end{gathered}$$

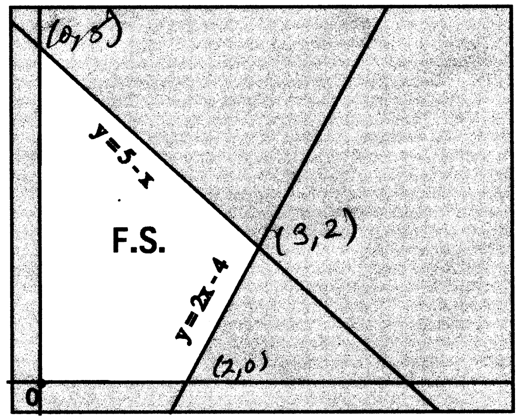
(b)
$$\begin{gathered} 5x + 3y \leq 120 \\ x \leq 3, \quad y \leq 5 \\ x \geq 10, \quad y \geq 15 \\ \text{Objective function: } 10x + 15y \end{gathered}$$

(c)
$$\begin{gathered} 10x + 15y \leq 120 \\ x \geq 3, \quad y \geq 5 \\ x \leq 10, \quad y \leq 15 \\ \text{Objective function: } 3x + 5y \end{gathered}$$

(d)
$$\begin{gathered} 10x + 5y \leq 120 \\ x \geq 3, \quad y \geq 5 \\ x \leq 10, \quad y \leq 15 \\ \text{Objective function: } 5x + 3y \end{gathered}$$

(e)
$$\begin{gathered} 5x + 3y \leq 120 \\ x \geq 3, \quad y \geq 5 \\ x \leq 10, \quad y \leq 15 \\ \text{Objective function: } 5x + 3y \end{gathered}$$

6 Find the maximum of the objective function, $5x + 10y$ on the feasible set drawn below.



(*a*) 35 (*b*) 50 (*c*) 8 (*d*) 7 (*e*) 55

Check objective function @ each

Vertex	$5x+10y$
(0,0)	0
(2,0)	10
(0,5)	50
(3,2)	35

$\boxed{\text{Max} = 50}$

Must Occur at corner pt.

1. (0,0)

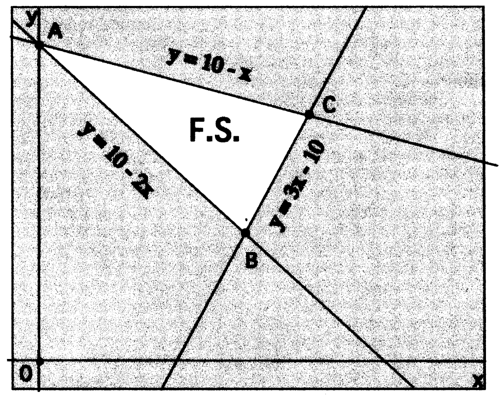
2. Where $y = 2x-4$ meets x-axis
   $2x-4=0 \Rightarrow x=2 \quad y=0$
   (2,0).

3. Where $y=5-x$ meets y-axis
   ie When $x=0 \rightarrow y=5$
   (0,5)

4. Where $y=5-x$ meets $y=2x-4$
   or $5-x = 2x-4$
   or $9 = 3x$
   $3 = x \Rightarrow y = 5-x = 5-3 = 2$
   (3, 2)

4

7 Find the vertices of the feasible set drawn below.



(a)	$A: x = 10,\ y = 0$	$B: x = 2,\ y = 4$	$C: x = 5,\ y = 5$
(b)	$A: x = 0,\ y = 10$	$B: x = 4,\ y = 2$	$C: x = 5,\ y = 5$
(c)	$A: x = 0,\ y = 10$	$B: x = 4,\ y = 5$	$C: x = 6,\ y = 7$
(d)	$A: x = 0,\ y = 10$	$B: x = 5,\ y = 2$	$C: x = 5,\ y = 5$
(e)	$A: x = 10,\ y = 0$	$B: x = 5,\ y = -1$	$C: x = 4,\ y = 2$

A: $y = 10 - x$ meets y-axis

$x = 0 \Rightarrow y = 10$

B $y = 10 - 2x$ meets $y = 3x - 10$

when $10 - 2x = 3x - 10$

or $20 = 5x$

or $4 = x \Rightarrow y = 10 - 2x = 2.$

C where $y = 3x - 10$ meets $y = 10 - x$

when $3x - 10 = 10 - x$

or $4x = 20$ or $x = 5$

$x = 5 \Rightarrow y = 10 - x = 5.$

5

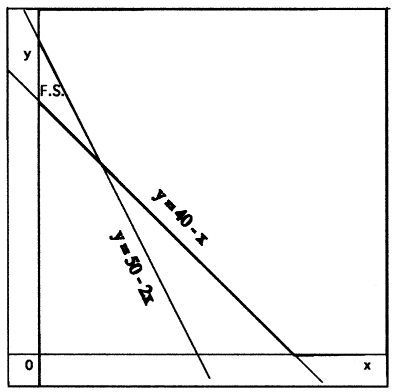
8 Which of the graphs shown below give the feasible set for the set of inequalities:

$$2x + y \geq 50 \qquad y \geq 50 - 2x$$

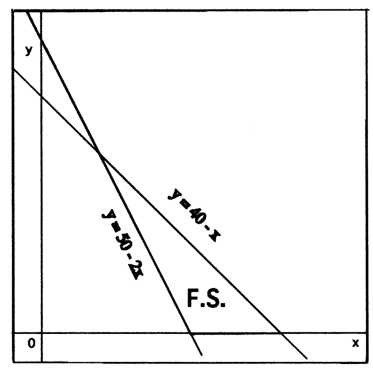
$$40 - y \leq x \qquad -y \leq x - 40 \rightarrow y \geq 40 - x.$$

$$y \geq 0, \quad x \geq 0$$

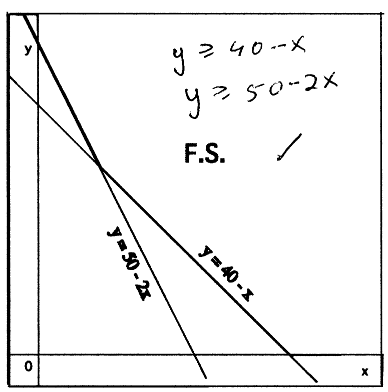
NOTE: The graphs are labeled (a) - (d) on the lower left.



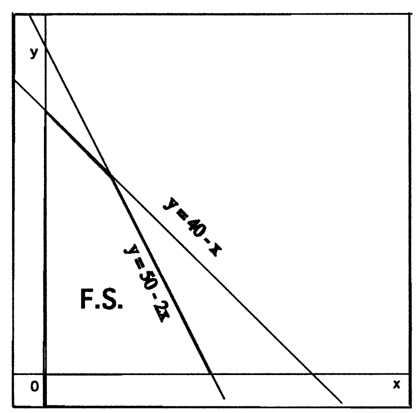
(a)



(b)



(c)



(d)

(e) None of the above.

6

9 What is the product $A \cdot B$ of the matrices:

$$A = \begin{pmatrix} 2 & 1 \\ 3 & 0 \\ -1 & 4 \end{pmatrix}, \qquad B = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}.$$

(a) $\begin{pmatrix} 5 & 6 & -2 \\ -1 & 0 & 4 \end{pmatrix}$ (b) $\begin{pmatrix} 4 & 1 \\ 0 & 0 \end{pmatrix}$ (c) $\begin{pmatrix} 7 & 2 \\ -3 & 0 \end{pmatrix}$

(d) $\begin{pmatrix} 5 & -1 \\ 6 & 0 \\ -2 & -4 \end{pmatrix}$ (e) $\begin{pmatrix} 4 & 1 \\ 6 & 3 \\ -2 & -5 \end{pmatrix}$

$$A \cdot B = \begin{pmatrix} 2 & 1 \\ 3 & 0 \\ -1 & 4 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 6 & 3 \\ -2 & -5 \end{pmatrix}_{3 \times 2}$$

10 Cromulus (C) and Remus (R) play a game in which they show one finger or two fingers, simultaneously. Cromulus then pays Remus an amount, in dollars, equal to the total number of fingers shown minus three dollars (If this sum is negative, it means that Remus pays Cromulus). What is Remus' (R's) pay-off matrix for this game?

(a)

# Fingers	C 1	C 2
R 1	0	−1
R 2	−1	−2

(b)

# Fingers	C 1	C 2
R 1	−1	0
R 2	0	1

(c)

# Fingers	C 1	C 2
R 1	2	1
R 2	−1	2

(d)

# Fingers	C 1	C 2
R 1	2	3
R 2	3	4

(e)

# Fingers	C 1	C 2
R 1	0	−1
R 2	1	0

$C \to R$.

Fingers R	C		
1	1	→	$1+1-3=-1$ Dollar
1	2	→	$1+2-3=0$
2	1	→	$2+1-3=0$
2	2	→	$2+2-3=1$

7

Part II, PARTIAL CREDIT, (10 Points each)

Show all of your work for credit

11, The grades for John for his freshman year are given below.

(a) Find John's average score (G.P.A.) for his freshman year.

4, 4, 4, 4, 3, 3, 2, 1, 1, 0

$$\frac{4+4+4+4+3+3+2+1+1+0}{10}$$

$$= \frac{16+9+4}{10} = \frac{29}{10} = 2.9.$$

(b) Calculate the variance σ^2 of John's scores

$(O_i-\bar{x})^2\frac{f_i}{n}$	O_i Out	f_i Freq	f_i/n Rel Freq	$(O_i-\bar{x})$	$(O_i-\bar{x})^2$
.841	0	1	.1	-2.9	8.41
.722	1	2	.2	-1.9	3.61
.081	2	1	.1	-.9	.81
.002	3	2	.2	.1	.01
.484	4	4	.4	1.1	1.21
$\sigma^2 =$ 2.13					

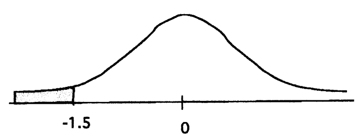
$\bar{x} = 2.9$

$$\sigma^2 = \sum (O_i - \bar{x})^2 \frac{f_i}{n}$$

8

12, The graphs below represent the standard normal distribution, i.e. the distribution of a random variable Z with mean $\mu = 0$ and standard deviation $\sigma = 1$. Find the area of the shaded region in each case.

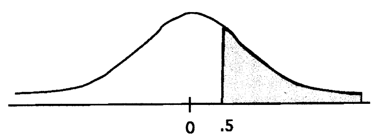
(a)



$Pr(Z \leq -1.5)$

$= .0668$

(b)



$Pr(Z \geq .5)$

$= 1 - Pr(Z \leq .5)$

$= .3085$

(c) The scores of the students who took the Math SAT exam are normally distributed with mean $\mu = 500$ and standard deviation $\sigma = 100$. What is the probability that a student chosen at random from among those tested had a score of 550 or more on the exam?

$Pr(X \geq 550)$

$= Pr\left(\frac{X - 500}{100} \geq \frac{550 - 500}{100}\right)$

$= Pr(Z \geq .5) = .0385$
From above.

9

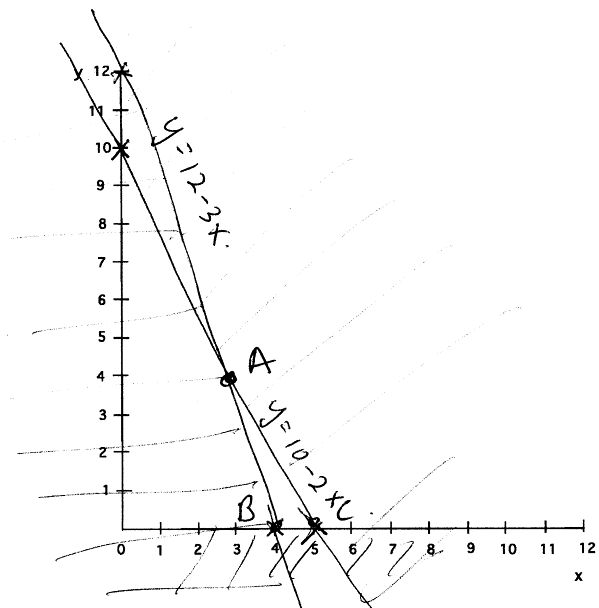
13, (a) Put the following set of inequalities in standard form, where relevant, and find the x and y intercepts of the line associated with each inequality, where relevant.

$$2x + y \leq 10$$
$$3x + y \leq 12$$
$$x \geq 0, \quad y \geq 0.$$

$y=0$ $\qquad$ $x=0$

Inequality	Standard form	Line	x-intercept	y-intercept
$2x + y \leq 10$	$y \leq 10-2x$	$y = 10-2x$	$x=5$.	$y=10$
$3x + y \leq 12$	$y \leq 12-3x$	$y = 12-3x$	$x=4$	$y=12$.
$x \geq 0$	$x \geq 0$	$x=0$		
$y \geq 0$	$y \geq 0$	$y=0$		

(b) Use the information from part (a) to graph the feasible set corresponding to the inequalities on the axes below. Mark the region corresponding to the feasible set F.S. and shade the regions not in the feasible set.



(c) Find the vertices of the above feasible set.

$y=10-2x$ meets $y=12-3x$.

$10-2x = 12-3x$

A: $x = 2$. $y = 10-4 = 6$

B. $x=4, y=0$

C $x=5, y=0$

10

14, Let

$$A = \begin{pmatrix} 1 & 2 \\ 0 & 1 \\ 2 & -1 \end{pmatrix}_{3\times 2}, \qquad B = \begin{pmatrix} -2 & 1 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{pmatrix}_{2\times 4} \qquad C = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}_{2\times 2}.$$

(a) In the following table, indicate which matrices can be calculated by writing Y if the matrix can be calculated and N if not. If the matrix can be calculated, then show the size of the resulting matrix in the third column. It is not necessary to calculate the matrix.

Matrix	Possible (Y/N)	Size ($m \times n$)
$A \cdot C$	Y	3×2.
$C \cdot A$	N	
$B \cdot C$	N	
$C \cdot B$	Y	2×4.
$A \cdot B$	Y.	3×4.
$B \cdot A$	N	

(b) Find the matrix C^{-1}, with the property that

$$C \cdot C^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = C^{-1}C$$

$$C = \begin{pmatrix} 1 & 2 \\ 1 & 3 \end{pmatrix}$$

$$C^{-1} = \frac{1}{1\cdot 3 - 1\cdot 2}\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$$

$$= \frac{1}{1}\begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 3 & -2 \\ -1 & 1 \end{pmatrix}$$

15 (a) Colm (C) and Rory (R) play a 2-player zero-sum game, where the payoff matrix for Rory is given by the following matrix:

	$C1$	$C2$	$C3$	MIN
$R1$	3	2	0	0
$R2$	−1	0	−2	−2
$R3$	0	4	−2	−2
MAX	3	4	0	

What is Rory's optimal fixed(pure) strategy for this game (Give the row number e.g R plays R3 on each play)?

R1

What is Colm's optimal fixed strategy for this game (Give the Column number)?

C3.

Does this pay-off matrix have a saddle point? If so where (give the address of the saddle point, e.g. (R2, C2))? Yes @ R1C3 Min in Row + Max in Col.

(b) Colleen (C) and Robert (R) play a 2-player zero-sum game, where the payoff matrix for Rasputin is given by the following matrix:

	$C1$	$C2$	$C3$	MIN
$R1$	−2	2	0	−2
$R2$	1	0	5	0
Max	1	2	5	

What is Robert's optimal fixed(pure) strategy for this game ?

R2

What is Colleen's optimal fixed strategy for this game?

C1.

Does this pay-off matrix have a saddle point? If so where ?

No.

(c) Coyote (C) and Roadrunner (R) play a 2-player zero-sum game, where the payoff matrix for Roadrunner is given by the following matrix:

	$C1$	$C2$	$C3$	MIN.
$R1$	2	1	4	1
$R2$	5	−1	3	−1
$R3$	1	−2	−5	−5
Max.	5	1	4.	

What is Roadrunner's optimal fixed(pure) strategy for this game ?

R1

What is Coyote's optimal fixed strategy for this game?

C2

Does this pay-off matrix have a saddle point? If so where?

Yes @ R1C2
Min in Row + Max in Col

12